

Quantum Field Theory without Infinite Renormalization

Tarun Biswas*

State University of New York at New Paltz,
New Paltz, NY 12561, USA.

(Dated: February 1, 2008)

Although Quantum field theory has been very successful in explaining experiment, there are two aspects of the theory that remain quite troubling. One is the no-interaction result proved in Haag's theorem. The other is the existence of infinite perturbation expansion terms that need to be absorbed into theoretically unknown but experimentally measurable quantities like charge and mass – i.e. renormalization. Here it will be shown that the two problems may be related. A “natural” method of eliminating the renormalization problem also sidesteps Haag's theorem automatically. Existing renormalization schemes can at best be considered a temporary fix as perturbation theory assumes expansion terms to be “small” – and infinite terms are definitely not so (even if they are renormalized away). String theories may be expected to help the situation because the infinities can be traced to the point-nature of particles. However, string theories have their own problems arising from the extra space dimensions required. Here a more directly physical remedy is suggested. Particles are modeled as extended objects (like strings). But, unlike strings, they are composites of a finite number of constituents each of which resides in the normal 4-dimensional space-time. The constituents are bound together by a manifestly covariant confining potential. This approach no longer requires infinite renormalizations. At the same time it sidesteps the no-interaction result proved in Haag's theorem.

PACS numbers: 03.70.+k, 11.10.-z, 11.10.Cd, 12.20.-m

I. INTRODUCTION

The infinities in perturbative Quantum Field Theories (QFT) can be traced to the point-nature of particles. If particles could be modeled as extended objects such infinities might disappear. String theories consider particles to be extended objects and hence they might solve the problem. However, string theories have problems of their own (for example, extra space dimensions). Renormalization schemes have worked, but the mathematics of a perturbation theory with expansion terms that are *infinite* is not satisfactory – even if these terms are absorbed into experimentally measured quantities like charge and mass. Here another attempt is made to model particles as extended objects – they are seen as composites of a finite number of constituent point objects. The fact that the constituents are once again point objects does not recreate the original problem of infinite perturbative terms because of some additional caveats that will be discussed soon. Each constituent resides in the usual physical 4-dimensional space-time. Composites of interacting point objects were considered to be inconsistent with relativity at one time[1]. However, later, more sophisticated mathematical techniques were discovered to deal with such composites in a relativistically covariant form[2, 3, 4, 5, 6]. A quantum field theory of such composites has been used to model hadrons as composites of quarks[7]. Here a similar formulation will be used to create composite models for particles that are traditionally not considered to be composites (electrons, photons,

etc.).

The proposed composite model of a particle has one *vertex* component with one or two *satellite* components. The vertex and the satellites are interacting point objects. A quantum field theory of such composites is a second quantized theory but has only first quantized interactions between satellites and the vertex. This mix of first and second quantization can be achieved quite seamlessly[7]. Some mathematical similarities exist between this model and string theories. However, this theory is significantly easier to visualize as it does not require objects to reside in anything other than normal 4-dimensional space-time. The vertex and satellites are somewhat like virtual particles of standard QFT. However, not being individually second quantized, they do not produce the ill-effects of an infinite number of them being created internally.

Experimentally observed effects like anomalous magnetic moment of electrons and the Lamb shift can be explained effectively by the dynamics of the satellites within particles. Hence, there is no need for closed loop diagrams of the particles themselves. As the closed loop diagrams are responsible for the offending infinities, eliminating them would eliminate the infinities. However, such diagrams are a natural consequence of perturbation theory. They appear due to the assumption that the free fields and the interacting fields are related by some unitary operator U_I . Now, the same assumption also leads to the no-interaction theorem of Haag[8]. So it is natural to suspect the unitarity of U_I . If U_I were not unitary, it would solve two problems at once. Here it is proposed that the effective part of U_I (call it V_I) be obtained by removing from U_I all contributions from closed loop diagrams. This will get rid of all infinities and at the same

*Electronic address: biswast@newpaltz.edu

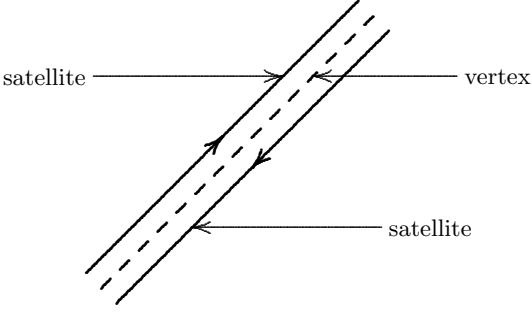


FIG. 1: Model of a boson

time render V_I nonunitary. Hence, the no-interaction problem of Haag will also be solved.

II. THE STRUCTURE OF BOSONS

In the present model, a bosonic particle is constituted of one spin-zero vertex and two spin-half satellites. In Feynman diagrams it will be represented as shown in figure 1. The dashed line represents the vertex and the satellites are represented by solid lines with arrows. The arrows suggest the fermion and anti-fermion nature of the satellites. The position and momentum four-vectors of the vertex are q_0 and p_0 [9]. The positions and momenta of the satellites are q_a and p_a ($a = 1, 2$). It is convenient to use coordinates relative to the vertex position and hence we define the following.

$$\begin{aligned} P &= p_0 + p_1 + p_2, \\ \pi_1 &= p_1, \\ \pi_2 &= p_2, \\ Q &= q_0, \\ \xi_1 &= q_1 - q_0, \\ \xi_2 &= q_2 - q_0. \end{aligned} \quad (1)$$

The resulting 24 dimensional phase space can be represented as

$$S_{p1} = \{P, \pi_1, \pi_2, Q, \xi_1, \xi_2\}, \quad (2)$$

where the only nonzero commutators are

$$[Q, P] = i\eta, \quad [\xi_1, \pi_1] = i\eta, \quad [\xi_2, \pi_2] = i\eta; \quad (3)$$

where η is the metric with signature $(-+++)$ and units are chosen such that $\hbar = 1$. It has been shown earlier[7] that S_{p1} can be canonically transformed to S_p given by

$$S_p = \{P, \pi_1^\parallel, \pi_1^\perp, \pi_2^\parallel, \pi_2^\perp, x, \xi_1^\parallel, \xi_1^\perp, \xi_2^\parallel, \xi_2^\perp\}, \quad (4)$$

where P and x are four-vectors representing the momentum and position of the particle as a whole. π_a^\parallel and ξ_a^\parallel are components of π_a and ξ_a parallel to P and π_a^\perp and ξ_a^\perp are projections of π_a and ξ_a orthogonal to P . These

components are obtained through the following projection operators.

$$-\hat{P} = -P/\sqrt{-P^2}, \quad P^\perp = \eta + \hat{P}\hat{P}, \quad (5)$$

where $P^2 = P \cdot P$ and “.” represents the usual four vector “dot” product. Note that \hat{P} has one suppressed index and P^\perp has two suppressed indices – the product in the definition of P^\perp is a tensor product. Then for any four-vector v we define

$$v^\parallel = -v \cdot \hat{P}, \quad v^\perp = v \cdot P^\perp. \quad (6)$$

v^\parallel is a scalar that is the zeroth component of v in the center of mass (CM) frame and v^\perp is a four-vector with no zeroth component in the CM frame. Hence, v^\perp is effectively a three-vector in the CM frame. It is to be noted that x is *not* the position of the vertex[7]. It is defined to remedy the problem of Q having complicated commutators with some of the other members of S_p . The result is the following set of non-zero commutators in S_p .

$$[x, P] = i\eta, \quad [\xi_a^\parallel, \pi_a^\parallel] = -i, \quad [\xi_a^\perp, \pi_a^\perp] = iP^\perp. \quad (7)$$

A maximal mutually commuting subset of S_p is

$$S_L = \{x, \xi_1^\parallel, \xi_1^\perp, \xi_2^\parallel, \xi_2^\perp\}. \quad (8)$$

Hence, the first quantized wavefunction of the system can be written as a function on S_L .

$$\psi = \psi(x, \xi_1^\parallel, \xi_1^\perp, \xi_2^\parallel, \xi_2^\perp). \quad (9)$$

The commutation conditions of equation 7 lead to the following differential operator representation of the momenta.

$$i(-\pi_a^\parallel, \pi_a^\perp) = \partial_{a\alpha} \equiv \left(\frac{\partial}{\partial \xi_a^\parallel}, \nabla_a \right), \quad (10)$$

and

$$iP_\alpha = \partial_\alpha \equiv \frac{\partial}{\partial x^\alpha}, \quad (11)$$

where α is the four-vector index and a the satellite number. The four-vector component notation (\dots, \dots) gives the zeroth component as the first argument and the three-vector components as the second argument. The three-vector operator ∇_a is defined to be the gradient in the three-vector space of ξ_a^\perp .

$$\nabla_a \equiv \left(\frac{\partial}{\partial \xi_{a1}^\perp}, \frac{\partial}{\partial \xi_{a2}^\perp}, \frac{\partial}{\partial \xi_{a3}^\perp} \right). \quad (12)$$

The wavefunction ψ must satisfy an equation of motion for each of the satellites and one for the vertex. The one for the vertex can be replaced by the whole particle equation. Hence, for free fields, it will be as follows.

$$D_0\psi \equiv (\partial_\mu \partial^\mu - m^2 - \mu^2) \psi = 0. \quad (13)$$

Here, m is the total rest mass of the particle. It includes all internal energies. The interaction energies of the satellites and the vertex are not physically separable. Hence, for mathematical convenience, an arbitrary separation is made such that the vertex energy in the CM frame is chosen as zero. Thus, the CM energy of the particle will be the sum of the CM energies of the two satellites alone. These can be seen to be the π_a^\parallel 's. However, the rest mass m of the particle is its CM energy. So,

$$m = \pi_1^\parallel + \pi_2^\parallel. \quad (14)$$

μ^2 is a constant parameter that physically tends to zero. However, it is needed in situations where $m = 0$. In such situations P^2 would be zero and hence \hat{P} would be ill-defined unless a positive μ^2 is included.

The equations of motion of the satellites cannot be expected to be free field equations. However, they can be made to *appear* like free spin-half fermion equations by including an interaction function in the mass term[10]. So, we write

$$D_a \psi \equiv (\gamma_a^\alpha \partial_{a\alpha} + m_a) \psi = 0, \quad (15)$$

where $a=1$ or 2 for the two satellites. γ_a^α is a Dirac matrix that operates on the sector of the wavefunction that corresponds to satellite number a . m_a is, in general, a function of all translation invariant members of S_p which excludes only x . However, further conditions must be imposed on m_a to maintain the consistency of the three equations of motion. This means the three operators D_0 , D_1 and D_2 must obey the following conditions.

$$[D_A D_B - D_B D_A] \psi = 0, \quad A, B = 0, 1, 2. \quad (16)$$

The simplest non-trivial way of satisfying equation 16 is to have m_a to be a function of ξ_a^\perp alone.

$$m_a = m_a(\xi_a^\perp). \quad (17)$$

This m_a acts as a potential energy function that keeps the satellites from escaping.

To obtain an orthonormal basis in the space of ψ one must define the following *universal current* as a generalization of conserved currents in field theories of structureless particles.

$$j^{\mu\alpha\beta} \equiv (i/2) \bar{\psi} \overleftrightarrow{\partial}^\mu \gamma_1^\alpha \gamma_2^\beta \psi, \quad (18)$$

where $\bar{\psi}$ is the adjoint of ψ and

$$\bar{\psi} \overleftrightarrow{\partial}^\mu \psi \equiv \bar{\psi} (\overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu) \psi \equiv \bar{\psi} (\partial^\mu \psi) - (\partial^\mu \bar{\psi}) \psi. \quad (19)$$

This yields the following conserved currents in each of the satellite sectors

$$j_1^\alpha \equiv \int j^{\mu\alpha\beta} d^3 x_\mu d^3 \xi_{2\beta}, \quad j_2^\beta \equiv \int j^{\mu\alpha\beta} d^3 x_\mu d^3 \xi_{1\alpha}, \quad (20)$$

where terms like $d^3 x_\mu$ represent four-vector hypersurface elements in x_μ space. The integrations are done over

arbitrary infinite spacelike hypersurfaces. A similar conserved current is obtained for the whole particle.

$$j^\mu \equiv \int j^{\mu\alpha\beta} d^3 \xi_{1\alpha} d^3 \xi_{2\beta}. \quad (21)$$

It is straightforward to prove the following conservation equations using the equations of motion[7].

$$\partial_{a\alpha} j_a^\alpha = 0, \quad a = 1, 2, \quad (22)$$

and

$$\partial_\mu j^\mu = 0. \quad (23)$$

All three conserved currents lead to the same conserved charge. It is given by

$$\mathcal{Q} \equiv \int j^{\mu\alpha\beta} d^3 \xi_{1\alpha} d^3 \xi_{2\beta} d^3 x_\mu. \quad (24)$$

Due to the conservation equations 22 and 23 it can be seen that the three integrations over space-like hypersurfaces are independent of the choice of any specific hypersurface. Hence, for convenience, we choose the $\xi_{a\alpha}$ hypersurfaces to be orthogonal to the total momentum P . This makes sure it is purely spatial in the CM frame. So, we replace $d^3 \xi_{a\alpha}$ by $d^3 \xi_a^\perp$ and γ_a^α by γ_a^\parallel where

$$\gamma_a^\parallel \equiv -\gamma_{a\alpha} \hat{P}^\alpha. \quad (25)$$

For the $d^3 x_\mu$ integration we choose the purely spatial components \mathbf{x} in the laboratory frame. Hence, $\overleftrightarrow{\partial}^\mu$ can be replaced by $\overleftrightarrow{\partial}^0$. This gives the conserved charge to be

$$\mathcal{Q} \equiv (-i/2) \int \bar{\psi} \overleftrightarrow{\partial}_0 \gamma_1^\parallel \gamma_2^\parallel \psi d^3 \xi_1^\perp d^3 \xi_2^\perp d^3 \mathbf{x}. \quad (26)$$

This conserved charge suggests the following natural norm for the Hilbert space of ψ .

$$(\psi, \psi) \equiv -i/2 \int \bar{\psi} \overleftrightarrow{\partial}_0 \gamma_1^\parallel \gamma_2^\parallel \psi d^3 \xi_1^\perp d^3 \xi_2^\perp d^3 \mathbf{x}. \quad (27)$$

This leads to the following definition of the inner product.

$$(\phi, \psi) \equiv -i/2 \int \bar{\phi} \overleftrightarrow{\partial}_0 \gamma_1^\parallel \gamma_2^\parallel \psi d^3 \xi_1^\perp d^3 \xi_2^\perp d^3 \mathbf{x}. \quad (28)$$

The above inner product definition allows us to identify the following orthonormal basis for the set of solutions of the equations of motion.

$$\begin{aligned} \psi_{\mathbf{k}E_1E_2} &\equiv [k^0(2\pi)^3]^{-1/2} \Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp) \cdot \\ &\cdot \exp[-iE_1 \xi_1^\parallel - iE_2 \xi_2^\parallel] \exp(ik \cdot x), \end{aligned} \quad (29)$$

where k is the four-vector eigenvalue of the total momentum P , \mathbf{k} is its three-vector part and k^0 is its zeroth component. E_a is the total energy of the a -th satellite. So it is the eigenvalue of π_a^\parallel . Note that $E_1 + E_2 = m$,

but it can be negative. This requires the usual explanation of an antiparticle being a particle going backward in time. So the physically measurable mass is still positive. For $\psi_{\mathbf{k}E_1E_2}$ to be a solution of the equations of motion in the satellite sectors, $\Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp)$ must satisfy the following eigenvalue equations for $a = 1, 2$.

$$H_a \Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp) = E_a \Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp), \quad (30)$$

where

$$H_a \equiv \alpha_a^\perp \cdot \pi_a^\perp + i\gamma_a^\parallel m_a, \quad \alpha_a^\perp \equiv -\gamma_a^\parallel \gamma_a^\perp, \quad (31)$$

with “ \parallel ” and “ \perp ” superscripts given the meaning of equation 6. It is to be noted that $\Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp)$ also depends on spin and orbital angular momentum quantum numbers due to rotational symmetry. The labels for these quantum numbers are suppressed for brevity of notation. Also, the spectrum for E_a is expected to be discrete as m_a produces a confining effect. For $\psi_{\mathbf{k}E_1E_2}$ to be a solution of the whole particle equation of motion, the following must be satisfied.

$$k^0 = \sqrt{\mathbf{k}^2 + (E_1 + E_2)^2}. \quad (32)$$

Ψ may be normalized in the usual fashion.

$$\int \bar{\Psi}(E'_1, E'_2, \xi_1^\perp, \xi_2^\perp) \gamma_1^\parallel \gamma_2^\parallel \Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp) d^3 \xi_1^\perp d^3 \xi_2^\perp = \delta_{E'_1 E_1} \delta_{E'_2 E_2}, \quad (33)$$

where $\delta_{E'E}$ is the Kronecker delta and, once again, the angular momentum labels are suppressed and understood to be included in the corresponding energy label. Using these conditions, it can be verified that the $\psi_{\mathbf{k}E_1E_2}$ are truly orthonormal.

$$(\psi_{\mathbf{k}'E'_1E'_2}, \psi_{\mathbf{k}E_1E_2}) = \delta_{E'_1 E_1} \delta_{E'_2 E_2} \delta(\mathbf{k}' - \mathbf{k}), \quad (34)$$

where $\delta(\mathbf{k}' - \mathbf{k})$ is the Dirac delta.

Now we are ready for second quantization. The standard prescription for canonical quantization will be used. However, it is critical to note that individual satellites and the vertex are not second quantized. It is the whole particle wavefunction ψ that is second quantized. The energies and angular momenta of the satellites are treated as extra degrees of freedom (quantum numbers) of the whole particle wavefunction.

First, a Lagrangian for the particle field is defined as follows.

$$\mathcal{L} = - \int \bar{\psi} \gamma_1^\parallel \gamma_2^\parallel [\partial_\mu^\leftarrow \vec{\partial}^\mu + m^2] \psi d^3 \xi_1^\perp d^3 \xi_2^\perp, \quad (35)$$

where m is given by equation 14. The momentum conjugate to ψ would then be

$$\phi \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} = -\bar{\psi} \gamma_1^\parallel \gamma_2^\parallel \partial_0^\leftarrow = \partial^0 \psi^\dagger, \quad (36)$$

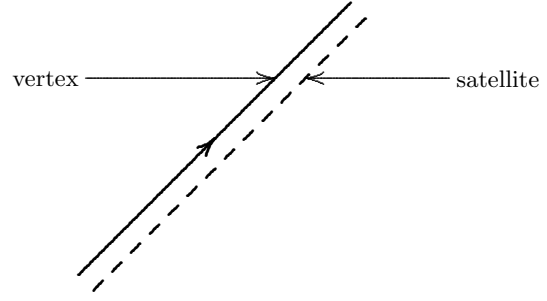


FIG. 2: Model of a fermion

where the Hermitian adjoint ψ^\dagger is related to the adjoint $\bar{\psi}$ as follows.

$$\psi^\dagger = \bar{\psi} (i\gamma_1^\parallel) (i\gamma_2^\parallel). \quad (37)$$

Then, from the spin-statistics theorem, one concludes that the second quantization condition can be written symbolically as the following equal-time commutator.

$$[\psi, \phi] = i\delta, \quad (38)$$

where the δ is a delta function over all degrees of freedom.

Now, ψ can be expanded in terms of the basis set of equation 29 as follows.

$$\begin{aligned} \psi = & \int d^3 \mathbf{k} \sum_{E_1 E_2} [2k^0 (2\pi)^3]^{-1/2} \Psi(E_1, E_2, \xi_1^\perp, \xi_2^\perp) \cdot \\ & \cdot \exp[-iE_1 \xi_1^\parallel - iE_2 \xi_2^\parallel] \cdot \\ & \cdot [b(\mathbf{k}, E_1, E_2) \exp(ik \cdot x) + \\ & + d^*(\mathbf{k}, E_1, E_2) \exp(-ik \cdot x)]. \end{aligned} \quad (39)$$

As in usual field theories, the b and d^* coefficients are used to separate particle and antiparticle states. d^* represents the Hermitian adjoint of d in a field operator sense. Then, the quantization condition of equation 38 reduces to the following (as before, the energy labels are understood to include angular momentum labels).

$$\begin{aligned} [b(\mathbf{k}, E_1, E_2), b^*(\mathbf{k}', E'_1, E'_2)] \\ = -[d^*(\mathbf{k}, E_1, E_2), d(\mathbf{k}', E'_1, E'_2)] \\ = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{E'_1 E_1} \delta_{E'_2 E_2}, \end{aligned} \quad (40)$$

and all other commutators of b , b^* , d and d^* vanish. This allows the building of the usual Fock space with b^* being the particle creation operator, b the particle annihilation operator, d^* the antiparticle creation operator and d the antiparticle annihilation operator. The necessary vacuum state can be shown to be stable[7].

III. THE STRUCTURE OF FERMIONS

A fermionic particle is also modeled as a composite. However, for this composite, the vertex is spin-half instead of spin-zero and there is only one satellite which is

spin-zero instead of spin-half. In Feynman diagrams it will be represented as shown in figure 2. The coordinates for the vertex and the satellite can be chosen in a fashion similar to bosons. The phase space S_p is as follows.

$$S_p = \{P, \pi^\parallel, \pi^\perp, x, \xi^\parallel, \xi^\perp\}, \quad (41)$$

where P and x are the momentum and position of the whole particle and π and ξ are the momentum and position of the spin-zero satellite. The non-zero commutators are as follows.

$$[x, P] = i\eta, \quad [\xi^\parallel, \pi^\parallel] = -i, \quad [\xi^\perp, \pi^\perp] = iP^\perp. \quad (42)$$

The wavefunction ψ is a function of a maximal mutually commuting subset of S_p as follows.

$$\psi = \psi(x, \xi^\parallel, \xi^\perp). \quad (43)$$

The commutation conditions of equation 42 lead to the following differential operator representation of the momenta.

$$i(-\pi^\parallel, \pi^\perp) = \partial_{1\alpha} \equiv \left(\frac{\partial}{\partial \xi^\parallel}, \nabla \right), \quad (44)$$

and

$$iP_\alpha = \partial_\alpha \equiv \frac{\partial}{\partial x^\alpha}. \quad (45)$$

∇ is the gradient operator in the three-vector space of ξ^\perp . The spin-half vertex equation of motion written as the whole particle equation should be akin to the Dirac equation. So, it is chosen to be as follows.

$$D_0\psi \equiv (\gamma^\alpha \partial_\alpha + m)\psi = 0, \quad (46)$$

where, as before, the CM energy of the vertex is chosen to vanish and hence the rest mass of the whole particle becomes the CM energy of the satellite.

$$m = \pi^\parallel. \quad (47)$$

The spin-zero satellite has an equation of motion as follows.

$$D_1\psi \equiv (\partial_{1\mu} \partial_1^\mu - m_1^2)\psi = 0, \quad (48)$$

where m_1 includes a confining potential for the satellite. So,

$$m_1 = m_1(\xi^\perp). \quad (49)$$

This form of m_1 ascertains the consistency of the equations of motion as seen in equation 16.

The universal current for fermions would then be given by

$$j^{\mu\alpha} \equiv (1/2)\bar{\psi} \overset{\leftrightarrow}{\partial}_1^\mu \gamma^\alpha \psi. \quad (50)$$

This helps define the conserved charge as

$$Q \equiv \int j^{\mu\alpha} d^3\xi_\mu d^3x_\alpha = (1/2) \int \bar{\psi} \overset{\leftrightarrow}{\partial}_1^\parallel \gamma^0 \psi d^3\xi^\perp d^3\mathbf{x}. \quad (51)$$

Then the inner product is seen to be

$$(\phi, \psi) \equiv 1/2 \int \bar{\phi} \overset{\leftrightarrow}{\partial}_1^\parallel \gamma^0 \psi d^3\xi^\perp d^3\mathbf{x}. \quad (52)$$

This allows the following orthonormal basis for the solution set of the equations motion.

$$\psi_{\mathbf{k}Er}^+ \equiv \sqrt{\frac{m}{Ek^0(2\pi)^3}} \Psi(E, \xi^\perp) \exp[-iE\xi^\parallel] u_r(k) \exp(ik \cdot x), \quad (53)$$

for positive energy states and

$$\psi_{\mathbf{k}Er}^- \equiv \sqrt{\frac{m}{Ek^0(2\pi)^3}} \Psi(E, \xi^\perp) \exp[-iE\xi^\parallel] v_r(k) \exp(-ik \cdot x), \quad (54)$$

for negative energy states. Here $u_r(k)$ and $v_r(k)$ are the usual free-field Dirac equation solutions with $r = \pm$ giving the two possible spin states. It is to be noted from equation 47 that $m = E$, as E is an eigenvalue of π^\parallel . Hence, the normalization factor above simplifies to $((2\pi)^3 k_0)^{-1/2}$. Also, k_0 is found to be

$$k^0 = \sqrt{\mathbf{k}^2 + E^2}, \quad (55)$$

and $\Psi(E, \xi^\perp)$ must satisfy the following eigenvalue equation.

$$H\Psi(E, \xi^\perp) = E\Psi(E, \xi^\perp), \quad (56)$$

where

$$H \equiv \sqrt{(\pi^\perp)^2 + m_1^2}. \quad (57)$$

Ψ can be normalized as follows.

$$\int \Psi^*(E', \xi^\perp) \Psi(E, \xi^\perp) d^3\xi^\perp = \delta_{E'E}. \quad (58)$$

Now we are ready to second quantize these fermion fields. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= -(1/2) \int \bar{\psi} \overset{\leftrightarrow}{\partial}_1^\parallel [\gamma^\alpha \partial_\alpha + m] \psi d^3\xi^\perp \\ &= \int \bar{\psi} \overset{\leftrightarrow}{\partial}_1^\parallel [\gamma^\alpha \partial_\alpha + m] \psi d^3\xi^\perp. \end{aligned} \quad (59)$$

So the momentum conjugate to ψ is

$$\phi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} = \partial_1^\parallel \bar{\psi} \gamma^0 = -i\partial_1^\parallel \psi^\dagger. \quad (60)$$

Symbolically, the second quantization condition is given by the following anticommutator.

$$\{\psi, \phi\} = i\delta. \quad (61)$$

All other anticommutators are zero. Anticommutators are used here in keeping with the spin-statistics theorem. In terms of creation and annihilation operators this relation can be written as

$$\{b(\mathbf{k}, E, r), b^*(\mathbf{k}', E', r')\} = \{d(\mathbf{k}, E, r), d^*(\mathbf{k}', E', r')\} = \delta^3(\mathbf{k} - \mathbf{k}')\delta_{E'E}\delta_{r'r}, \quad (62)$$

where b and d are defined as the following expansion coefficients of ψ in terms of the orthonormal basis.

$$\psi = \int d^3\mathbf{k} \sum_E \sum_r [b(\mathbf{k}, E, r)\psi_{\mathbf{k}Er}^+ + d^*(\mathbf{k}, E, r)\psi_{\mathbf{k}Er}^-]. \quad (63)$$

All anticommutators of b , b^* , d and d^* other than the ones in equation 62 vanish.

IV. INTERACTION CENTERS

The composite particle models described in the last two sections can be generalized further. One may have a spin-zero vertex with any number of spin-half as well as spin-zero satellites. Similarly, a spin-half vertex may have any number of spin-zero or spin-half satellites. As a result, the photon, the W^\pm , the Z and all mesons can be modelled as bosonic composites while the electron, the muon, the tau, all neutrinos and all baryons can be modelled as fermionic composites.

However, at present, I shall consider only electromagnetic interactions. The photon is a composite boson and the electron a composite fermion. The fermionic satellites of a free photon will have energies that are equal in magnitude but opposite in sign. This gives the photon a zero rest mass. The following interaction Lagrangian can describe all experimentally observed interactions.

$$\mathcal{L}_I = e \int \bar{\psi}_a(x, \xi_1) [d^3\xi_1^\perp \overleftarrow{\partial}_1^\parallel] \gamma_{ab}^\mu \bar{\phi}_{b\bar{c}}(x, \xi_1, \xi_2) \cdot \gamma_{c\bar{d}\mu} [d^3\xi_2^\perp \overrightarrow{\partial}_2^\parallel] \psi_d(x, \xi_2), \quad (64)$$

where ψ is used to represent fermion fields and ϕ for boson fields. The spinor indices are written explicitly and indices like \bar{a} are used to label adjoint spinor indices. Indices like a and \bar{a} used here are not to be confused with the satellite number index used in section II. Contractions are over a and \bar{a} , b and \bar{b} etc.. $\bar{\phi}$ is the adjoint of ϕ for only the second satellite sector. Hence, \bar{c} is used to denote the second spinor index. e is the electron charge. The integration variables ξ_1^\perp and ξ_2^\perp are coordinates orthogonal to the corresponding *electron* momenta and not the photon momentum.

In Feynman diagrams the effects of \mathcal{L}_I will be called the interaction *centers* instead of *vertices* as they are usually called. This is because the name *vertex* has been used for a component of a composite particle. Figure 3 shows the diagram for such an interaction center. Note

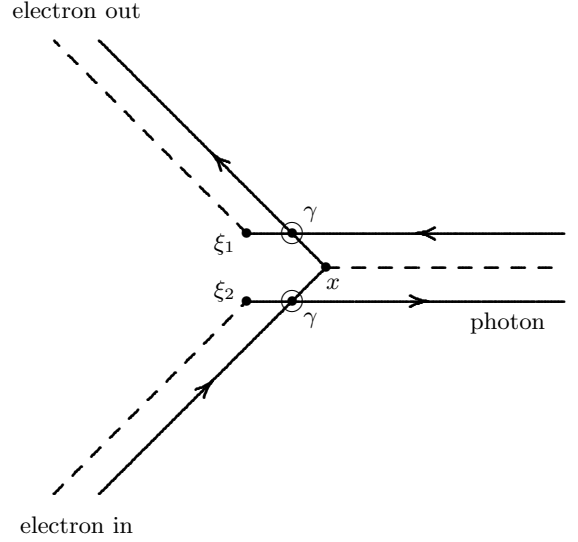


FIG. 3: Interaction center for QED.

that the diagram shows a coupling of the coordinates of the two satellites of the photon to the satellites of the two electrons. This accurately describes \mathcal{L}_I as shown in equation 64. Equation 64 also shows a coupling of spins of each photon satellite to each of the two electron vertices. They are mediated by the two γ_μ matrices. This is represented in the diagram by the intersections of the fermion lines. In usual field theories, it is known that fermion lines do not begin or end at an interaction center. Here, that is still true. However, little pieces of the fermion lines extend out from the intersection points (marked by circles) to represent their couplings to bosonic components. One way of visualizing this is to consider the photon to be forming a fermionic current due to its fermionic satellites. It is given by

$$A_{b\bar{d}\mu}(x, \xi_1, \xi_2) = \bar{\phi}_{b\bar{c}}(x, \xi_1, \xi_2) \gamma_{c\bar{d}\mu}. \quad (65)$$

The current of the electron fields would be

$$J_{d\bar{b}}^\mu(x, \xi_1, \xi_2) = \bar{\psi}_a(x, \xi_1) \overleftarrow{\partial}_1^\parallel \gamma_{a\bar{b}}^\mu \overrightarrow{\partial}_2^\parallel \psi_d(x, \xi_2). \quad (66)$$

Then the interaction can be seen as an interaction of two currents:

$$\mathcal{L}_I = e \int J_{d\bar{b}}^\mu(x, \xi_1, \xi_2) A_{b\bar{d}\mu}(x, \xi_1, \xi_2) d^3\xi_1^\perp d^3\xi_2^\perp. \quad (67)$$

At the same time, it is also noticed that $A_{b\bar{d}\mu}(x, \xi_1, \xi_2)$ is the composite particle analog of the usual vector field $A_\mu(x)$ representing photons.

At first sight, the above choice of an interaction center may seem arbitrary. However, closer inspection shows that there aren't too many other choices. Some of the natural restrictions on \mathcal{L}_I are as follows.

- It must be Lorentz invariant.

- It must be a natural extension of the free field Lagrangians of both photons and electrons.
- It must be symmetric in the two electron lines.
- It must agree with all experimentally verified results of usual QED.
- No fermionic line (satellites or vertex) can end at the interaction center.
- The photon field should not appear as a pure gauge.

The following is a slight variation of \mathcal{L}_I that satisfies all but one of the above restrictions.

$$\mathcal{L}_{I0} = e \int \bar{\psi}_a(x, \xi_1) [d^3 \xi_1^\perp \overset{\leftarrow}{\partial}_1^\parallel] \gamma_{ab}^\mu \bar{\phi}_{cd}(x, \xi_1, \xi_2) \cdot \gamma_{d\bar{c}\mu} [d^3 \xi_2^\perp \overset{\rightarrow}{\partial}_2^\parallel] \psi_b(x, \xi_2). \quad (68)$$

Here the spinor indices of the photon satellites couple with each other and the two electron spins couple with each other. This form of the interaction will not be discussed any further here as it does not produce the experimentally observed anomalous electron magnetic moment. It only produces a *finite* charge renormalization. Hence, it can be ignored.

V. PERTURBATION THEORY AND HAAG'S THEOREM

Using the interaction described by equation 64 (or 68) one can find scattering matrix elements for QED interactions. However, only tree diagrams need be considered. The effects of closed loops in diagrams are completely included in the interaction center. So, instead of infinite loop integrals we will have integrals over internal coordinates like ξ_1^\perp and ξ_2^\perp . If the field operators are expanded in terms of the basis sets (equations 29, 53 and 54), these integrals will produce summations over the bound state energies (and angular momenta) of the satellites. As these energies are discrete, the summations are likely to produce finite results. This is qualitatively similar to Planck's derivation of the blackbody radiation spectrum – a summation over discrete energies produced the correct finite result instead of the infinite result of Raleigh-Jeans obtained by integration over continuous energies.

Physically, this makes sense as the virtual particles of loop diagrams are like internal bound particles. So, their states should be expected to display a discrete spectrum. But in traditional QED there is no way of recognizing this feature and hence, it produces infinite integrals for closed loops in diagrams. In the composite particle QED formulated here, this discreteness appears naturally and hence, it is likely to produce finite results.

However, strict adherence to perturbation theory requires the use of the interaction picture. This will still

produce closed loop diagrams of composite particles. So, here we need to deviate from the interaction picture as follows.

Of all possible Feynman diagrams produced by the interaction picture, only the tree diagrams will be considered physical.

This assumption does not weaken the theory in any way as the effects of the closed loop diagrams are included in the internal structure. At the same time, it avoids infinite renormalizations. It also has an unintended, but highly desirable, side effect – Haag's theorem[8] is no longer an impediment. Let U_I be the usual unitary operator that transforms free fields to interacting fields. In the present formulation, we are keeping only tree diagrams. This means we are keeping only a part of U_I – say V_I . This V_I will not be unitary and hence, Haag's theorem is no longer valid in this formulation.

VI. QED – AGREEMENT WITH EXPERIMENT

To find experimental consequences of the present formulation, we shall compute the magnetic moment of the electron. For lowest order effects we compute the interaction energy of an electron and a photon field while all satellites are in their ground states. Let the incoming electron have a momentum p , the outgoing electron a momentum p' and the incoming photon a momentum q . Then, integrating \mathcal{L}_I of equation 64 over d^4x produces the Dirac delta $\delta(p' - p - q)$. Hence,

$$q = p' - p. \quad (69)$$

The remaining part of the integral is the interaction energy of interest. It is

$$H_I = \bar{u}_a(p') \gamma_{ab}^\mu W_b \bar{W}_c \gamma_{cd} u_d(p). \quad (70)$$

where $\bar{u}_a(p')$, and $u_d(p)$ are Dirac spinor components for the outgoing and incoming free electrons. Also,

$$W_b = \int \Psi^*(E_0, \xi_1^\perp) \Psi_b(e_0, \xi_1^{\perp'}) d^3 \xi_1^\perp, \quad (71)$$

$$\bar{W}_c = \int \Psi(E_0, \xi_2^\perp) \bar{\Psi}_c(e_0, \xi_2^{\perp'}) d^3 \xi_2^\perp. \quad (72)$$

Here E_0 is the ground state energy of the electron satellite and e_0 that of each of the two photon satellites. Ψ is the electron satellite ground state wavefunction and Ψ_b the ground state wavefunction of one of the two photon satellites. Ψ_b is obtained by separating the photon Ψ of equation 30 into the two separate satellite wavefunctions. The ξ_1^\perp are components of ξ_1 in a hypersurface orthogonal to the corresponding electron momentum. The $\xi_1^{\perp'}$ are components of ξ_1 in a hypersurface orthogonal to the corresponding photon momentum. To compute the integrals of equations 71 and 72, one needs the mass functions

of equations 17 and 49. Explicit forms for these functions will be introduced in a later article. For now, it is to be noticed that one may write

$$W_b \bar{W}_{\bar{e}} \gamma_{c\bar{d}\mu} = (I_{b\bar{d}} + \Delta_{b\bar{d}}) a_\mu(q), \quad (73)$$

where $I_{b\bar{d}}$ is the identity for Dirac spinors and $a_\mu(q)$ can be interpreted as the momentum representation of the usual electromagnetic vector potential for a momentum q . The $\Delta_{b\bar{d}}$ provides the anomalous magnetic moment. For this to agree with experiment in lowest order one must have

$$\Delta_{b\bar{d}} = \frac{i\alpha}{4\pi E_0} \gamma_{b\bar{d}\nu} q^\nu, \quad (74)$$

where α is the fine structure constant. A complete computation of $\Delta_{b\bar{d}}$ will be presented in a later article.

VII. CONCLUSION

A quantum field theory of composite electrons and photons is suggested. It is distinctly different from string theories in many ways - in particular, it does not require space-time to have dimensions greater than four. It also does not require infinite renormalizations. As a side-effect, the no-interaction result of Haag's theorem is avoided as well. Experimentally testable aspects of usual QED (anomalous electron magnetic moment, Lamb shift etc.) are expected to be reproduced in this theory.

-
- [1] D. G. Currie, T. F. Jordan and E. C. G. Sudarshan, *Rev. Mod. Phys.*, **38**, 350 (1963).
 - [2] R. Arens, *Nuovo Cimento B*, **21**, 395 (1975).
 - [3] Ph. Droz-Vincent, *Rep. Math. Phys.*, **8**, 79 (1975).
 - [4] V. V. Molotov and I. T. Todorov, JINR Report EZ-12270, Dubna (1979).
 - [5] A. Komar, *Phys. Rev. D*, **18**, 1881, 1887 (1978).
 - [6] T. Biswas, *Nuovo Cimento A*, **88**, 154 (1985).
 - [7] T. Biswas, *Nuovo Cimento A*, **107**, 863 (1994).
 - [8] R. F. Streater and A. S. Wightman, *PCT, Spin*

and Statistics, and All That, pp. 165-166 (Benjamin/Cummings, 1980).

- [9] the four-vector indices are suppressed for compactness.
- [10] This is the conceptual equivalent of the one-dimensional wave equation of string theories. However, it is more general, as it allows for a large class of binding potentials in three space dimensions. String theories effectively use only the one-dimensional particle in a box potential.